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Investigation of Inflow Performance Relationship in Gas Reservoirs for Vertical and Horizontal Wells

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Abstract

Horizontal wells have been applied all over the world because of their high productivity. The well performance of these wells has not been well-defined yet. Therefore, the main objectives of this study are to evaluate the well performance of horizontal wells and compare it to that one of vertical wells. Using the same drainage areas and similar fluid properties, the well productivity and Inflow Performance Relationship (IPR) for both vertical and horizontal wells are evaluated and compared for steady-state flow of compressible and incompressible fluids. Current models for both types of vertical and horizontal wells are evaluated to stress their strengths and weaknesses. The replacement ratio of horizontal well to vertical well are calculated. Furthermore, a sensitivity analysis is performed on common variables to compare and evaluate vertical and horizontal well flow equations.

Keywords: well performance; gas reservoir; vertical well; horizontal well

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1. INTRODUCTION

An inflow performance relationship (IPR) relates the well production rate as a function of the drawdown pressure and gives a comprehensive understanding of what the reservoir can deliver into the well at a specific time [1].

The inflow performance of horizontal and vertical wells is characterised by different IPRs. Bendakhlia and Aziz [2] (1989) showed that using an IPR developed for a vertical well gave unsatisfactory results for horizontal well flow which should have its own specifically derived IPR. Furui, Zhu and Hill [3] (2003) also noted that the drainage pattern and flow geometry of horizontal and vertical wells were different. A horizontal well was more likely to have radial flow near the wellbore and linear flow away from the wellbore while a vertical well was most likely to have radial flow only, highlighting the need for separate IPRs.

Horizontal wells are becoming increasingly popular and economically viable with technological improvements [4] and new analytical equations and correlations are constantly being developed in order to fully characterise reservoir performance. A quantitative comparison using various new models and correlations as well as industry standards should be performed in order to determine the model which best describes steady state flow in both horizontal and vertical wells. Moreover, as the difference between the cost and performance of a vertical or horizontal well in the same reservoir will be very different [5] well orientation is often a difficult decision faced by many companies. To aid in this, a replacement ratio of horizontal well to vertical well will be calculated. A sensitivity analysis of key fluid and rock properties common to the models will also be examined in order to determine and quantify the dominating factor to the calculation of both horizontal and vertical IPRs.

This study investigates steady-state flow in horizontal and vertical wells for compressible flow. Single and multi-phased compressible and incompressible fluids in horizontal and vertical steady-state flow are characterised quite differently, as the reservoir behaves differently in each flow regime. Single-phased flow IPRs are characterised using analytical methods for both vertical and horizontal flow [6]. These analytic formulae are vital for predicting the productivity of both horizontal and vertical wells and aid the decision making process and development of a reservoir [4]. Two-phased flow however, is characterised by correlations rather than analytical methods because of complexities which include the treatment of relative permeability [7] and composition and phase change which occurs with reservoir depletion, which are not easily modelled [8].

1.1 Background Theory

Numerous literatures is available on the IPRs for both horizontal and vertical wells with much of the literature focused on the creation of analytical models or correlations to model horizontal well productivity under particular reservoir and flow conditions. As the use of horizontal wells became more common, studies on horizontal well deliverability and the formation of horizontal IPRs have also become more widespread with industry standards such as Joshi [9] (1988) and Cheng [10] (1990) coming under examination [11]. Vertical wells however, have been well established within the industry for much longer and a great body of work has been reported on the calculation of a vertical IPR [6]. For example the equation derived by Vogel equation [12] (1968) for vertical, two phased flow, has become an accepted industry standard for inflow performance calculation and so is one of the models chosen for comparison.

Well productivity estimation is still a challenge, especially with two-phased flow [13] and models calculating horizontal and vertical IPRs are frequently being produced to better describe reservoir flow. These models should be under constant revision and compared with those commonly used within industry. This is performed by Kamkom and Zhu [11] (2006) which looked at "Generalised Horizontal Well Inflow Relationships for Liquid, Gas or Two-Phase Flow" and adjusted various correlations in order to better describe reservoir flow. The same procedure will be applied and the adjusted correlations by Kamkom and Zhu [11] (2006) and others will be compared to those commonly used in industry.

1.1.1 Productivity of Horizontal and Vertical Wells

The comparison of the productivity of horizontal and vertical wells has been reported by several authors, each with specific reservoir conditions. For a thin oil zone, Kossack and Kleppe [5] (1987) concluded that a horizontal well exhibited much better performance than a conventional vertical well if the horizontal well length was more than 1500 ft.

Fleming [14] (1993) compared the performance of vertical and horizontal IPRs using data from a reservoir within the Piceance Basin of Colorado. Hashemi and Gringarten [13] (2005) also had similar results when they compared the production of a horizontal well to that of a non-stimulated vertical well in a gas-condensate reservoir found that horizontal wells increase productivity in dry gas systems and enhance productivity even further below the dew point.

Dashti, Mar and Kabir [15] (2001) however, found that a horizontal does not always offer higher productivity and looked at the high permeability and high anisotropy Burgan Third Middle Sand reservoir in Kuwait where production was tubing-constrained, meaning that “deliverability at the sandface overwhelmed that at the surface, regardless of orientation”. This sentiment is also expressed by Mukherjee and Economides [16] (1991).

Although studies quantifying the difference between horizontal and vertical IPRs for a particular field or area are useful, they cannot be applied to all reservoirs across the field. Instead, they are applicable only if the properties of reservoir in question match those of the reservoir studied. This problem is addressed by Mukherjee and Economides [16] (1991). They examined several scenarios including comparing a fully completed horizontal well with a fractured vertical well in a low permeability reservoir and the performance of a hydraulically fractured horizontal well to that of a hydraulically fractured vertical well. Mukherjee and Economides (1991) concluded that horizontal wells are preferable to vertical wells in most cases assuming an idealized vertical isotropic medium. However, in a reservoir with reasonable vertical anisotropy ($I_{ani} > 1.5$) and low permeability (≤ 0.1 md) a hydraulically fractured vertical well is preferable.

All of the above-reviewed studies assumed that the pressure gradient though the horizontal part of a horizontal well was negligible in order to simplify their theoretical models. In reality, this is not the case and can be seen in production logs as reported by Folefac et al [17] (1991) who found that this assumption often led to the over prediction of the productivity index and deliverability of horizontal wells. This issue is also addressed by Shedid and Zekri [18] (2005) who calculated horizontal well performance experimentally, thereby eliminating the assumption of a negligible pressure gradient along horizontal wells in common theoretical models.

1.1.2 Effect of Well and Rock/Fluid Properties

1.1.2.1 Vertical Wells with Compressible Fluid

The IPR for a vertical well depends on the number of phases present either for compressible (a gas) or slightly compressible (water and oil).

The steady-state relationship based on Darcy’s law [21] for an incompressible fluid can be adjusted for a compressible fluid, by using an average gas formation volume factor

$$\bar{B}_g = \frac{0.0283\bar{Z}T}{\left(p_e + p_{wf}\right)/2} \tag{1}$$

This average gas formation volume factor is a function of both pressure and temperature, resulting in a Darcy's gas well deliverability of

$$p_e^{\ 2} - p_{wf}^{\ 2} = \frac{1424 q \bar{\mu} \bar{Z} T}{kh}\left(\ln \frac{r_e}{r_w} + s\right) \tag{2}$$

This approximation is only acceptable for small gas flow rates as Equation (2) assumes that only Darcy flow occurs [1]. Equation (2) is often written as

$$q = C\left(\bar{p}^2 - p_{wf}^{\ 2}\right) \tag{3}$$

Fetkovich [[22]] (1973) showed that where non-Darcy flow was evident, such as for large flow rates, Equation (3) should be adjusted to

$$q = C\left(\bar{p}^2 - p_{wf}^{\ 2}\right)^n \tag{4}$$

where 0.5 < n <1 and is determined by fitting the data on a logarithmic curve [19] (Akhimiona & Wiggins, 2005)

A more accurate model to characterise the gas deliverability of a vertical gas well was developed by Aronofsky and Jenkins [23] (1954) who utilised the Forchheimer equation of flow and developed the time-dependant IPR

$$q = \frac{kh\left(\bar{p}^2 - p_{wf}^{\ 2}\right)}{1424 \bar{\mu} \bar{Z} T\left[\ln\left(r_d / r_w\right) + s + Dq\right]} \tag{5}$$

Where D is the non-Darcy flow coefficient and r_d is the effective drainage radius as defined by Aronofsky and Jenkins [[23]] (1954) as

$$\frac{r_d}{r_w} = 1.5\sqrt{t_D} \tag{6}$$

Where t_D is the time dependence of the relationship and is

$$t_D = \frac{0.000264 kt}{\phi \mu c_t r_w^{\ 2}} \tag{7}$$

Note that Equation (5) is only time dependant until $r_d = 0.472 r_e$. The non-Darcy flow coefficient, D can be approximated by the empirical relationship found in Economides, Hill and Ehlig-Economides [1] (1994)

$$D = \frac{6 \times 10^{-5} \gamma k_s^{\ -0.1} h}{\mu r_w h_{perf}^{\ 2}} \tag{8}$$

Where k_s is the near wellbore permeability in md, γ is the gas gravity, h and h_{perf} are the net and perforation thicknesses in feet and μ is the gas viscosity in cP.

1.1.2.2 Horizontal Wells with Compressible Fluid

Horizontal wells are considered most effective in thin reservoirs as they increase the wellbore contact area with the reservoir, reservoirs with good vertical permeability and those which have water or gas coning problems [20].

The IPR for horizontal wells differ from that of vertical wells with two major differences. Firstly, flow regimes and secondly anisotropy. These added differences make horizontal well performance more difficult to determine analytically.

The IPR of a horizontal gas well is commonly found by adjusting the model used to find the horizontal oil well deliverability as presented in Kamkom and Zhu [11] (2006) where Furui, Zhu and Hill's model [3] (2003) for gas wells is adjusted to take into account the varying formation volume factor which is a function of pressure and temperature, and the non-Darcy flow effects due to the high velocity flow usually typical of gas wells.

$$q = \frac{7kL\left(p_e^{\ 2} - p_{wf}^{\ 2}\right)}{1424\overline{Z\mu_g}T\left[\ln\left(\frac{hI_{ani}}{r_w\left(I_{ani}+1\right)}\right) + \frac{\pi y_b}{hI_{ani}} - 1.224 + s\right]} \tag{9}$$

Note that the gas viscosity and the gas compressibility are average values which are taken at average pressure.

Akhimiona and Wiggins [19] (2005) analysed the pressure rate performance of horizontal gas wells using a three-dimensional finite difference reservoir simulator for various reservoir and wellbore conditions and then fit a curve to the data to obtain an IPR. They found that plotting the data in terms of pressure-squared gave the best coefficient of fit with Equation 10 giving a concave curve

$$\frac{q_g}{q_{g,\max}} = 1 - 1.867\left(\frac{p_{wf}^{\ 2}}{\bar{p}^2}\right) + 0.867\left(\frac{p_{wf}^{\ 2}}{\bar{p}^2}\right)^2 \tag{10}$$

2. SIMULATOR DESCRIPTION

The software package used Petroleum Production Systems (PPS) is a comprehensive software package which aims to "aid production engineers in performing design and diagnosis for oil and gas well production" [25]. The software package is primarily based on theory presented in the book, *Petroleum Production Systems*, by Economides, Hill, and Ehlig-Economides [1] (1994) but also includes calculations from other sources.

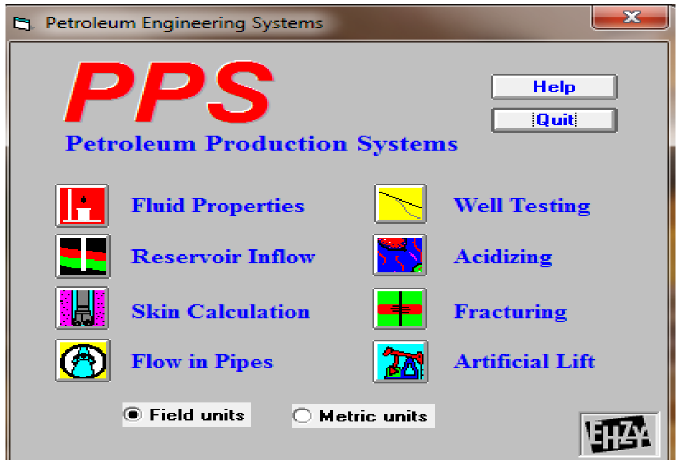

Figure 1: The PPS opening screen box with modules

The PPS software package contains eight modules as shown in Figure 1 which are

Fluid Properties	Reservoir Inflow	Skin Calculation
Flow in Pipes	Well testing	Acidizing
Fracturing	Artificial Lift	

The gas IPR can be calculated using a pseudo-pressure relationship, m(p_{wf}), or a pressure-squared, $(p_{wf})^2$ relationship. For consistency, the pressure squared relationship was chosen as this is the form that pressure takes in both our vertical and horizontal analytic models.

2.1. Evaluations and Comparison of the Performance of Vertical and Horizontal Wells

The evaluation of horizontal and vertical wells is made using the analytical models and empirical correlations. Petroleum Production Systems (PPS), a comprehensive petroleum software package is also used to calculate IPRs for comparison. The IPRs calculated by both the models and PPS will be compared using Microsoft Excel, a simple spreadsheet program.

2.1.1 Base Case

Base case simulations and calculations were performed using the default input data values given by the PPS software with all variable used for each case. Base case simulations were run for vertical and horizontal wells in

2.1.1.1 Vertical Well Base Cases with a Compressible Fluid

Darcy's analytical solution which models Darcy flow in a gas well and Aronofsky and Jenkins' [23] (1954) equation which models non-Darcy flow is compared to both Darcy and non-Darcy flow in PPS as shown in Figure 2. Both Darcy's [21] and Aronofsky and Jenkins' [23] models require the assumption of an average viscosity for the entirety of the reservoir. Darcy's equation also requires the assumption of an average compressibility factor, both of which would introduce an element of error into the calculations. In order to calculate the average viscosity, empirical correlations by Carr, Kobayashi and Burrows [24] (1954) were used. The average compressibility factor for the base case gas reservoir was found using an empirical correlation developed by Standing and Katz [26] (1942).

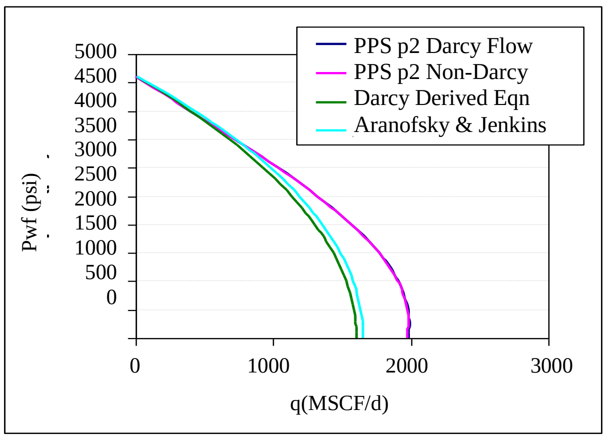

Figure 2: Comparison of vertical gas well IPRs

Aronofsky and Jenkins' [23] equation also requires the calculation of the perforations height which was assumed to be half of the reservoir net thickness. On comparison, Darcy's model in Equation (2) and Aranofsky and Jenkins' model in Equation (5) are identical except for the non-Darcy flow factor, *Dq*, contained in Aranofsky and Jenkins' equation. This is reflected in Figure 2 with both Darcy's model and Aranofsky and Jenkins' model giving very similar flow rates, with Aranofsky and Jenkins giving slightly higher flow rates, consistent with it's inclusion of the non-Darcy flow factor, *Dq*.

It is also noted in Figure 2 that, the IPRs from the PPS simulator are significantly higher than those calculated using analytical solutions.

2.1.1.2 Horizontal Well Base Cases with a Compressible Fluid

The horizontal well base case in a gas reservoir is compared and computed using Kamkom and Zhu's [7] equation, Akhimiona and Wiggins' [19] equation and the results of the PPS simulator with non-Darcy flow.

To calculate an IPR for a horizontal gas well, PPS uses an adjusted Joshi's equation for a compressible fluid which is given by

$$q = \frac{k_H h\left(p_e{}^2 - p_{wf}{}^2\right)}{1424\mu ZT\left[\ln\left(\frac{a+\sqrt{a^2-(L/2)^2}}{(L/2)}\right)+\frac{I_{ani}h}{L}\left\{\ln\left(\frac{I_{ani}h}{r_w\left(I_{ani}+1\right)}\right)+Dq\right\}\right]} \tag{11}$$

This Equation (also incorporates the non-Darcy flow coefficient, *D* which takes into account the effects of turbulence. Joshi's equation for a horizontal gas well was also used in Akhimiona and Wiggins' [19] equation as it requires the calculation of the absolute open flow (AOF).

A comparison of these models is shown in Figure 3, where a great deal of variation in the IPRs calculated can be seen. Differing models give entirely different flow rates, highlighting the inconsistencies of differing models in characterising horizontal gas flow. Figure also shows that the curvature of Akhimiona and Wiggins' [19] equation is not entirely concave which is typical of all other IPRs but instead, goes from convex at pressures close to initial pressure to concave at AOF.

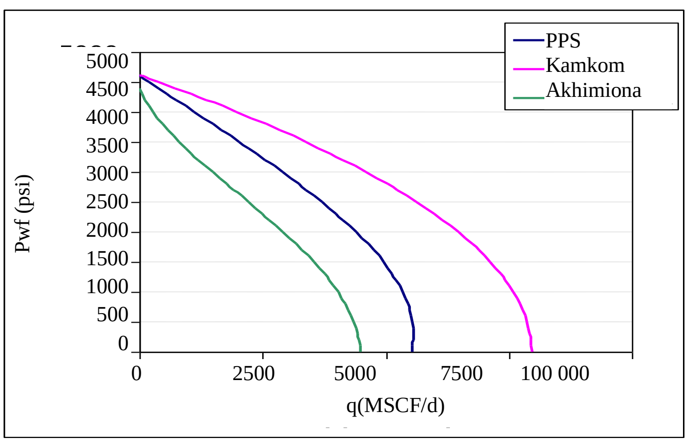

Figure 3: Comparison of horizontal gas well IPRs

When modelling a horizontal gas well, the turbulence effects can be treated as negligible. This is because as seen in Equation (11), the turbulence effects, *Dq*, is multiplying by the scaling ratio $I_{ani}h/L$, reducing the pressure drop from the near-wellbore turbulence [1].

It can also be seen that the flow rate of the horizontal well is far greater than the flow of the vertical well in the base cases gas reservoir. This is generally the case for most reservoirs but there will be a length at which the productivity from a vertical well will meet that of a horizontal well in the base case reservoir.

2.1.1.3 Evaluation of Key Well, Fluid and Rock Variables

A sensitivity analysis was performed on the key well, fluid and rock properties which horizontal well length (L), oil viscosity (μ), formation thickness (h), formation isotropy ($\sqrt{k_V / k_H}$) and skin (s)

Each variable was changed in certain domain and an IPR calculated in order to study the magnitude of the change on the vertical and horizontal IPR models, with the appropriate base case used as a standard benchmark.

The models used in the base case simulation were again employed to calculate IPRs in the sensitivity analysis, changing the variables listed above, over the range shown in Table 1. For comparison, the AOF for each IPR calculated were plotted for each model to better compare the behaviour of the models in the sensitivity analysis. This is a valid way of comparing different models as the sensitivity analysis has shown that the inherent curvature of each IPR is maintained, independent of the variable that is changed. In this way, the IPR for each model can be summarised in a single plot in either a horizontal or vertical well, making comparison of the models much easier.

Table 1: Base case values and range of values for sensitivity analysis

	Parameter	Base Case	Cases Investigated
Gas	Horizontal Well Length (ft)	1000	100 to 3000
	Anisotropy	3	0.1 to 4
	Viscosity (cP)	0.0241	0.005 to 0.09
	Skin	0	-7 to 7
	Formation thickness (ft)	78	50 to 450

2.1.1.4 Horizontal Well Length in an Gas Well

The horizontal well length was varied from 100 to 3000 ft. Changing the length of the horizontal region of the wellbore only affects flow in horizontal wells. The horizontal wellbore length is varied for each model, with the general trend of an increase in horizontal wellbore length gives an increase in production displayed by all curves. The shape of the IPR was also found to be independent of wellbore length for all fluid types, which is consistent with finding by Akhimiona and Wiggins [19] (2005).

Varying the horizontal wellbore length results in different behaviour with different models used characterising horizontal gas flow as seen in Figure 4. Kamkom and Zhu's [7] equation can be seen to give a linear relationship which can be attributed to the equation seen in Equation (9). Kamkom and Zhu's [7] equation has wellbore length, L, located on the numerator and so an increase in wellbore length will always correspond to a proportional increase in flow rate.

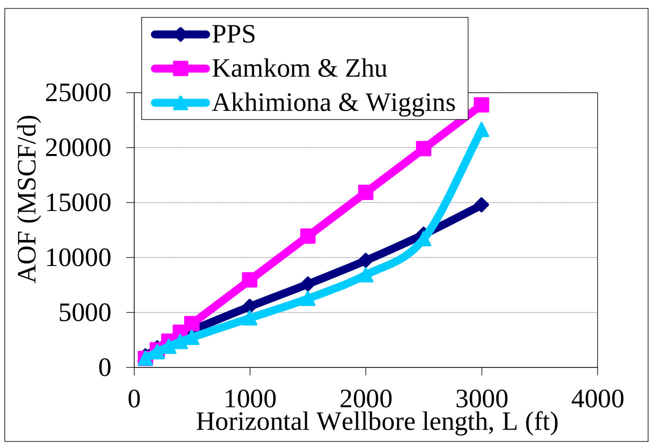

Figure 4: Horizontal gas AOF with varying horizontal wellbore length

This is contrasted to Akhimiona and Wiggins' model [19] and PPS which shows a non-linear relationship which has a decreasing AOF until a wellbore length of 500 ft and then an increasing AOF with increasing wellbore length. As an empirical model, Akhimiona and Wiggins' equation requires the calculation of the AOF, $q_{o,max}$ using an equation which models horizontal gas flow. This is done using Joshi's equation for horizontal gas flow as given in Equation (11) which is also employed by PPS to calculate a horizontal gas IPR, explaining why both models have similar behaviour. It can be seen in Joshi's equation for horizontal gas flow that the wellbore length, L, does not have a linear relationship with flow rate, resulting in a non-uniform increase or decrease in the AOF with a regular increase in horizontal wellbore length.

When examining results of Akhimiona and Wiggins' [19] equation in Figure 4, an unusually large increase in AOF when increasing the horizontal wellbore length from 2000 ft to 3000 ft can be clearly noted. Although seemingly large, the value given by Akhimiona and Wiggins is consistent with the value given by Kamkom and Zhu's equation but is much higher than that calculated by PPS.

2.1.2 Gas Viscosity

Gas viscosity was varied between 0.005 – 0.09 cP. This range was determined using gas viscosity empirical correlations by Carr, Kobayashi and Burrows [24] (1954). This required the calculation of pseudo-critical and pseudo-reduced temperature and pressures and for ease of calculation, we assume the reservoir pressure and gas gravity was constant. This means that we have a constant pseudo-critical pressure and pseudo-reduced pressure as well as a constant composition. Therefore, the corresponding temperature for each viscosity was found and is given in Table 2.

Table 2: The temperatures and viscosity ranges used in this study

μ	0.005	0.01	0.0241	0.03	0.04	0.05	0.06	0.07	0.08	0.09
$\mu/\mu 1_{atm}$	0.526	0.908	1.96	2.3	3.00	3.649	4.2	4.8	5.3	5.8
$\mu 1_{atm}$	0.0095	0.011	0.0123	0.0127	0.0132	0.0137	0.0142	0.0146	0.015	0.0154
T	40	100	180	210	240	270	300	330	360	390

The values in Table 2: were then used to calculate IPRs both horizontal and vertical gas wells. The AOF values for different models used to describe vertical oil flow can be seen in Figure 5. It can be seen that both Aronofsky and Jenkins' and Darcy's models for vertical gas flow give the expected exponential relationship with decreasing viscosity but PPS does not. It can be concluded that although the calculated temperatures and pressures correspond to the same viscosities, changing the reservoir temperature does not give the same expected exponential behaviour. This is because back-calculating a change in viscosity to a change in temperature and pressure introduces large amounts of error into the calculation.

Figure 5 shows that both Darcy's and Aronofsky and Jenkins' [23] models give very similar results. When examining the models, we find that Aronofsky and Jenkins' equation gives AOF values approximately 50 MSCF/day higher than those calculated by Darcy for viscosity values above the base case viscosity of $\mu = 0.0241$ cP. For viscosities less than $\mu = 0.0241$ cP, Darcy's flow giving much higher AOF values than Aronofsky and Jenkins' equation. From the results of varying viscosity alone, we are unable to determine whether either Darcy's equation or Aronofsky and Jenkins' equation is more accurate.

The same exponential relationship between viscosity and flow rate is also seen when modelling flow in a horizontal gas well as shown in Figure 6 with the exception of the AOF rates calculated by PPS. This is again due to the error introduced when back-calculating from viscosity to temperature and pressure. Although the same exponential relationship can be seen with both Kamkom and Zhu [11] and Akhimiona and Wiggins' [19] models they give very different AOF rates. The AOF given by Kamkom and Zhu is much higher than those calculated by Akhimiona and Wiggins' equation with the difference increasing at lower viscosities, highlighting the variance in results given by different horizontal models.

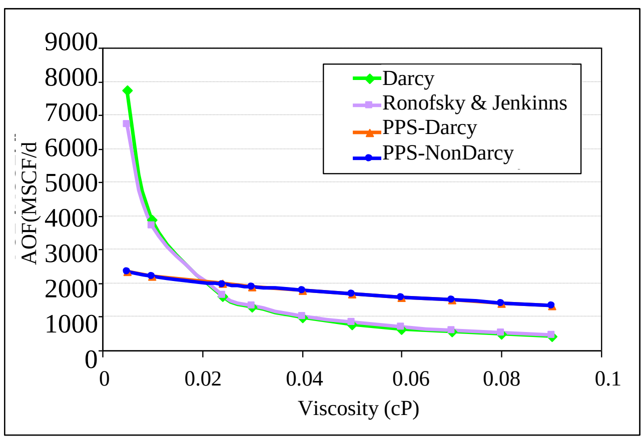

Figure 5: Vertical gas AOF with varying viscosity

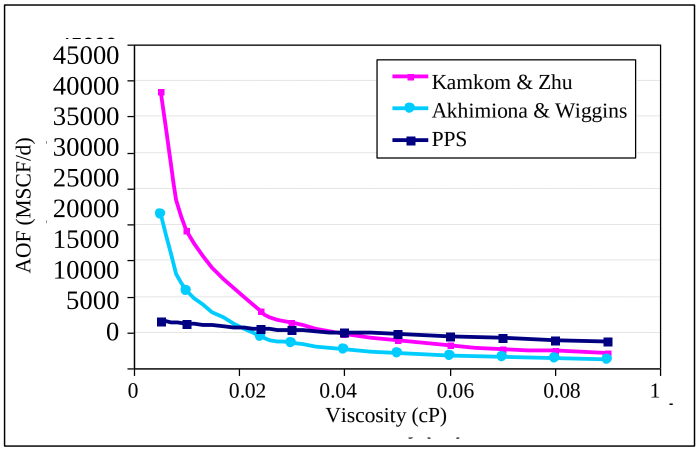

Figure 6: Horizontal gas AOF with varying viscosity

2.1.2.1 Formation Thickness in an Gas Reservoir

For all cases in all fluids, reservoir formation thickness was varied between 50 and 450 ft. An increase in formation thickness increases the pay zone and the area open to flow, corresponding to an increase in IPR and therefore AOF. This hypothesis is consistent with all results, independent of fluid type.

Using Darcy's equation, Aronofsky and Jenkins' [23] equations and PPS to calculate vertical gas IPRs gives results as shown in Figure 7. Similar to Darcy's equation for oil flow, all models used to describe vertical gas flow show a positive linear relationship between formation thickness and the AOF. Note the AOF calculated by Aronofsky and Jenkins is slightly higher than those given by Darcy's method as they take into account the non-Darcy component of gas flow.

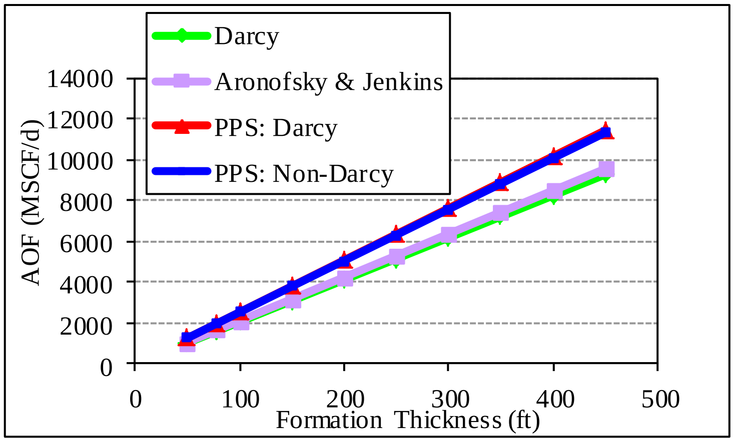

Figure 7: Vertical Gas AOF with varying formation thickness

The PPS gives higher flow rates with both Darcy and non-Darcy flow than the flow rates calculated using analytical equations. In all cases, the linearly proportional relationship between flow rate and formation thickness in vertical gas flow can be attributed to a linear relationship in the relevant equation.

A horizontal well in a gas reservoir does not give the same linearly proportion behaviour with varying formation thickness as seen for a vertical gas well. Instead, as formation thickness increases, flow rate increases in decreasing increments which can be seen by all models when comparing the AOF in Figure 8.

The flow rates calculated by Akhimiona and Wiggins' [19] and PPS plateau at approximately 300 ft, which suggests that after this thickness, an increase in formation thickness does not significantly increase the flow rate or productivity of the well. Although PPS gives consistently higher flow rates than those given by Akhimiona and Wiggins' equation, they are of the same magnitude with values close enough to each other to make either method preferable to Kamkom and Zhu's [7] equation for calculating the IPR for horizontal well in a gas reservoir. This is because there is a huge discrepancy in AOF rates calculated between Kamkom and Zhu's equation and both Akhimiona and Wiggins' equation and PPS with Kamkom and Zhu giving unrealistically high flow rates. This occurs at all formation thickness but becomes more pronounced as formation thickness increases.

2.1.2.1 Anisotropy Ratio in an Gas Reservoir

In order to vary the anisotropy ratio, it is assumed that vertical permeability, kV, is constant at the base case value and vary horizontal permeability kH. Although the permeability anisotropy ratio is the same for all fluid types, as permeability varys for a gas reservoir and oil or two-phase reservoir two separate tables of permeability cases are required. For gas flow, Table 3 includes anisotropy and permeability values used in gas IPR equation.

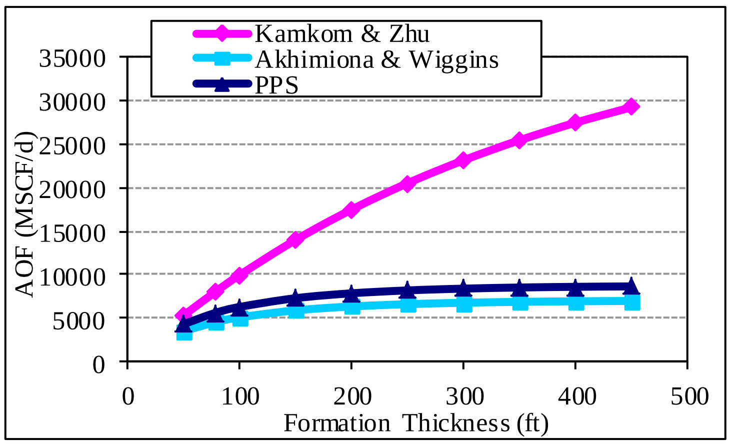

Figure 8: Horizontal gas AOF with varying formation thickness

Table 3: Range of permeability anisotropy ratios used

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	Case 10
Iani	0.50	1.00	1.50	2.00	2.50	3.02	4.00	5.00	7.00	9.07
kH	0.225	0.9	2.025	3.6	5.625	8.2	14.4	22.5	44.1	74
kV	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
k	0.45	0.90	1.35	1.80	2.25	2.72	3.60	4.50	6.30	8.16

For a vertical gas well, both Darcy's and Aronofsky and Jenkins' equations employ permeability, *k*, to calculate IPRs This results in a positive linear relationship between both Darcy's and Aranofsky and Jenkins' equations and anisotropy ratio is evident in Figure 9. Note that in order to vary the anisotropy ratio, the permeability values from Table 3 are used.

When comparing Aronofsky and Jenkins' equation to Darcy's equation for a vertical gas IPR, it can be seen that both have a similar flow rate with Aronofsky and Jenkins having slightly higher AOF values with a maximum difference of approximately 50 STB/d. This is most likely due to the effect of non-Darcy flow. The results given by PPS for Darcy and non-Darcy flow are non-linear as seen in Figure 9. As previously seen in both the base case and sensitivity analysis, the flow rates calculated by PPS are significantly higher than those calculated using analytical equation.

The AOF results of different models for a horizontal gas well can be shown in Figure 10. Again, different models give different flow rates and trends with varying anisotropy ratio.

Kamkom and Zhu's [7] model in Figure 10 is observed to be linear relationship with flow rate, but upon closed inspection and examining Kamkom and Zhu's equation in Equation (9), a slight difference in flow rate can be seen as the anisotropy ratio changes.

A stronger non-linear relationship can be seen by both PPS and Akhimiona and Wiggins' [19] equation. As seen in Figure , at low anisotropy ratios PPS and Akhimiona and Wiggins' equation gives similar flow rates but as anisotropy ratio increases, the flow rates diverge. This may be because both PPS and Akhimiona and Wiggins' equation employ Joshi's equation for horizontal gas flow in Equation 11 to calculate flow rate.

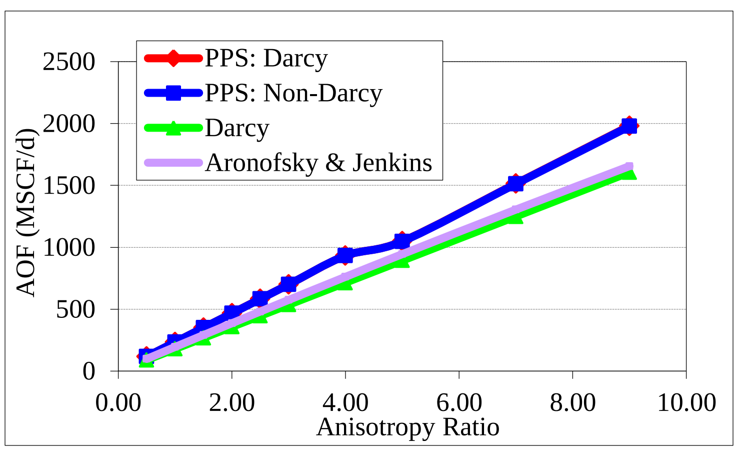

Figure 9: Vertical gas AOF with varying anisotropy ratio

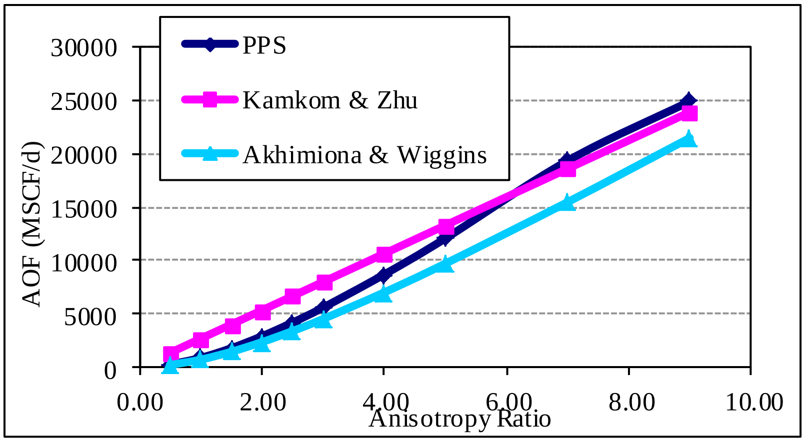

Figure 10: Horizontal gas AOF with varying anisotropy ratio

It should also be noted that although all the models describing horizontal gas flow employ anisotropy ratio, they also use another closely related variable in the numerator of the IPR equation. For example, Kamkom and Zhu's equation uses permeability, k, on the numerator whereas Joshi's equation for horizontal gas flow which is used both PPS and Akhimiona and Wiggins' uses horizontal permeability, k_H, on the numerator which could also be a factor in the differing flow rates.

2.1.2.3 Skin in a Gas Reservoir

Skin is a dimensionless number which represents a pressure drop in the near-wellbore region and is usually caused by a distortion of the flow lines or a restriction to flow. One way of distorting the flow-lines is through damage to the reservoir's natural permeability and therefore the effects of a positive skin can be seen to be similar to the effects of a reduction in permeability. A negative skin however, means that the pressure drop in the near well-bore region is smaller than normal and improves flow. In this paper, skin is varied for all cases between -7 and +7.

The skin effect in a horizontal well, s'_{eq}, is characteristic of the shape of damage in horizontal wells and takes into account the permeability anisotropy and the likelihood of larger damage penetration nearest to the vertical section [1].

The IPR of a vertical gas well with varying skin can be calculated using Darcy's model Aronofsky and Jenkins' model and PPS and are the AOF of which are shown in figure 11. All models show approximately the same behaviour of an increasing flow rate as skin decreases. For positive skin values, Aronofsky and Jenkins' model giving slightly higher flow rates than Darcy's model with a maximum difference of approximately 50 MSCF/day. At large negative skin values (s = -7), Darcy's equation gives 65% higher results. PPS however, gives flow rates which consistently give higher values than both analytic models.

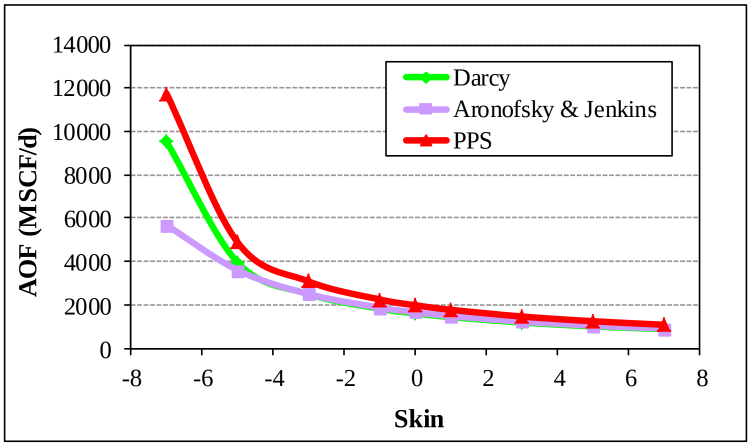

Figure 11: Vertical gas AOF with varying skin

For horizontal gas flow, both PPS and Akhimiona and Wiggins' equation utilise Joshi's equation for horizontal gas flow in Equation (11) which in its original form does not account for skin effects. However, Economides, Hill & Ehlig-Economides [1] (1994) includes the effect of skin by adding the damage skin effect within the second set of brackets in the denominator as shown below in Equation (12)

$$q = \frac{k_H h\left(p_e{}^2 - p_{wf}{}^2\right)}{1424\mu ZT\left[\ln\left(\dfrac{a+\sqrt{a^2-(L/2)^2}}{(L/2)}\right)+\dfrac{I_{ani}h}{L}\left\{\ln\left(\dfrac{I_{ani}h}{r_w\left(I_{ani}+1\right)}\right)+Dq+s'_{eq}\right\}\right]} \tag{12}$$

The modified Joshi's equation is then employed by both PPS and Akhimiona and Wiggins' [[19]] model for calculating flow in a horizontal gas well.

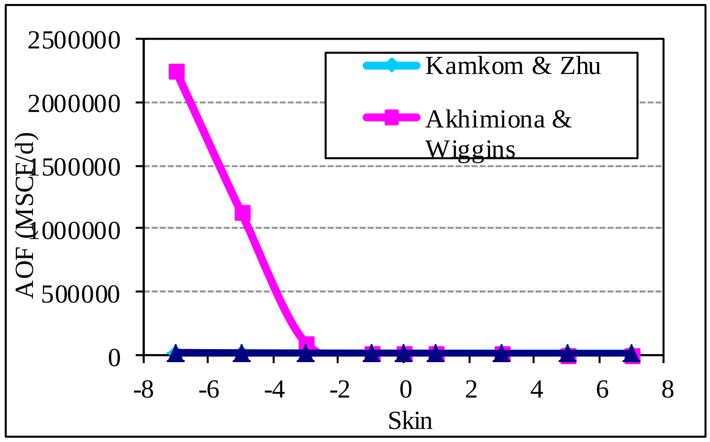

Figure 12: Horizontal gas AOF with varying skin

It is immediately obvious from figure 12 that Akhimiona and Wiggins' [19] equation model gives very large flow rates for large negative skin values. For example, Akhimiona and Wiggins' equation gives a maximum flow rate of over 2 million MSCF/day for a skin value of, s = -7 which for the inputs used, is and, for the variables used, can be seen to be unrealistic. From this we can conclude that Akhimiona and Wiggins' equation is only applicable for positive skin and low values of negative skin.

The large flow rates given by Kamkom and Zhu's [11] equation can be eliminated by removing large negative skin values and plotting

Figure over a reduced range as seen in Figure 12. For positive skin values and low negative skin values, it can be seen that the three models still generate very different results. Kamkom and Zhu's equation gives an approximately linear relationship between flow rate and skin over a 2000 MSCF/d range. PPS follows a similar shape, with slightly more curvature but gives lower flow rates whereas Akhimiona and Wiggins' model gives an approximately exponential relationship over a much larger range of flow rates.

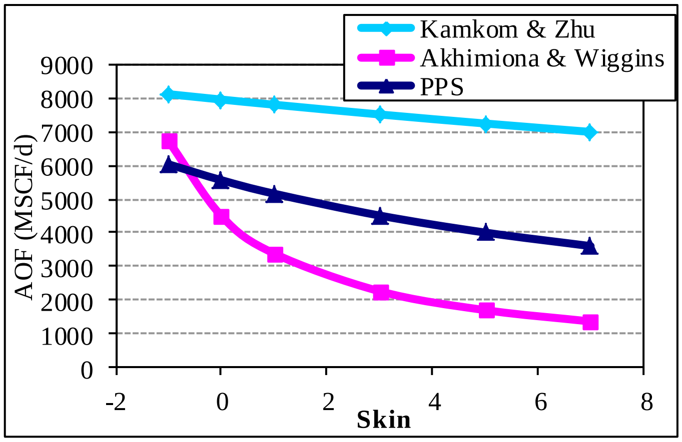

Figure 13: Horizontal gas AOF over a reduced range of skin

The AOF results for a horizontal gas well with varying skin again differs when different models are used, once more highlighting the inability of current flow models to accurately characterise horizontal flow.

3. CONCLUSIONS

A comprehensive evaluation of inflow performance relationship (IPR) for both vertical and horizontal well have been achieved and the following conclusions are drawn:

- All models of compressible and incompressible fluid under steady-state flow condition have been evaluated and compared for both vertical and horizontal wells.
- The vertical and horizontal wells' analysis performed has highlighted the inconsistency of current models in characterising horizontal well flow.
- The effects of flowing viscosity, reservoir anisotropy, formation thickness, horizontal well length, skin factor, and other parameters have been evaluated.
- The proposed reservoir model has been found to have severe effect on the predicted IPR results for horizontal wells and has no effect for vertical ones.
- The rock anisotropy has shown an influential impact on resultas values for both vertical and horizontal wells.

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